Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **May/June 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

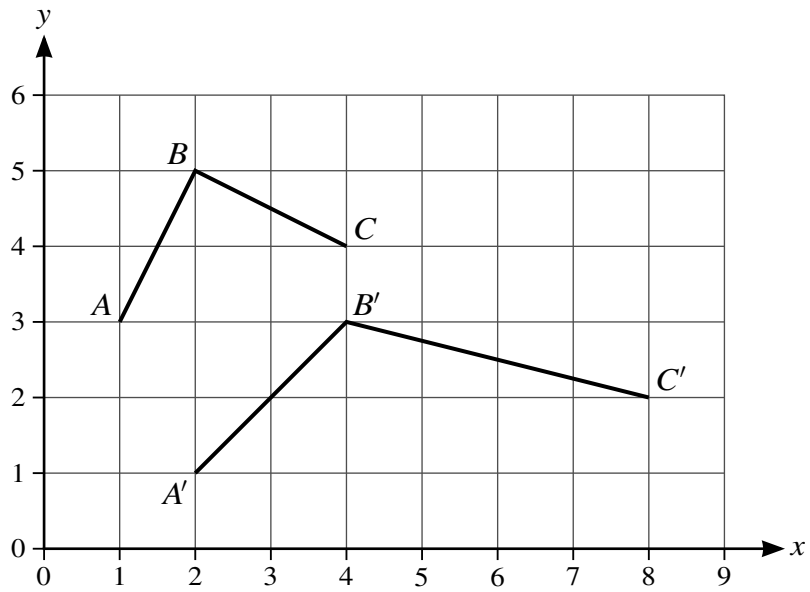
INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

[Turn over

1



The diagram shows the graph of $y = f(x)$, which consists of the two straight lines AB and BC. The lines $A'B'$ and $B'C'$ form the graph of $y = g(x)$, which is the result of applying a sequence of two transformations, in either order, to $y = f(x)$.

State fully the two transformations. [4]

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2 The function f is defined for $x \in \mathbb{R}$ by $f(x) = x^2 - 6x + c$, where c is a constant. It is given that $f(x) > 2$ for all values of x.

Find the set of possible values of c. [4]

3 **(a)** Give the complete expansion of $\left(x + \frac{2}{x}\right)^5$. [2]

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(b) In the expansion of $(a + bx^2)\left(x + \frac{2}{x}\right)^5$, the coefficient of x is zero and the coefficient of $\frac{1}{x}$ is 80.

Find the values of the constants a and b. [4]

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4 **(a)** Show that the equation

$$3\tan^2 x - 3\sin^2 x - 4 = 0$$

may be expressed in the form $a\cos^4 x + b\cos^2 x + c = 0$, where a, b and c are constants to be found. [3]

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(b) Hence solve the equation $3\tan^2 x - 3\sin^2 x - 4 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

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5 A circle has equation $(x - 1)^2 + (y + 4)^2 = 40$. A line with equation $y = x - 9$ intersects the circle at points A and B.

(a) Find the coordinates of the two points of intersection. [4]

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(b) Find an equation of the circle with diameter AB. [3]

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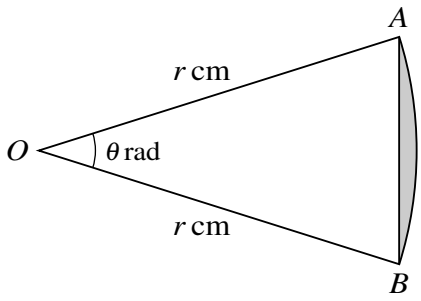

The diagram shows a sector OAB of a circle with centre O and radius r cm. Angle $AOB = \theta$ radians. It is given that the length of the arc AB is 9.6 cm and that the area of the sector OAB is 76.8 cm^2.

(a) Find the area of the shaded region. [5]

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(b) Find the perimeter of the shaded region. [2]

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7 The function f is defined by $\mathrm{f}(x) = 2 - \dfrac{5}{x+2}$ for $x > -2$.

(a) State the range of f. [1]

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(b) Obtain an expression for $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [4]

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The function g is defined by $g(x) = x + 3$ for $x > 0$.

(c) Obtain an expression for $fg(x)$ giving your answer in the form $\frac{ax+b}{cx+d}$, where a, b, c and d are integers. [3]

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8 A progression has first term a and second term $\frac{a^2}{a+2}$, where a is a positive constant.

(a) For the case where the progression is geometric and the sum to infinity is 264, find the value of a. [5]

(b) For the case where the progression is arithmetic and $a = 6$, determine the least value of n required for the sum of the first n terms to be less than -480. [5]

9 A curve which passes through (0, 3) has equation $y = f(x)$. It is given that $f'(x) = 1 - \frac{2}{(x-1)^3}$.

(a) Find the equation of the curve. [4]

The tangent to the curve at (0, 3) intersects the curve again at one other point, P.

(b) Show that the x-coordinate of P satisfies the equation $(2x + 1)(x - 1)^2 - 1 = 0$. [4]

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(c) Verify that $x = \frac{3}{2}$ satisfies this equation and hence find the y-coordinate of P. [2]

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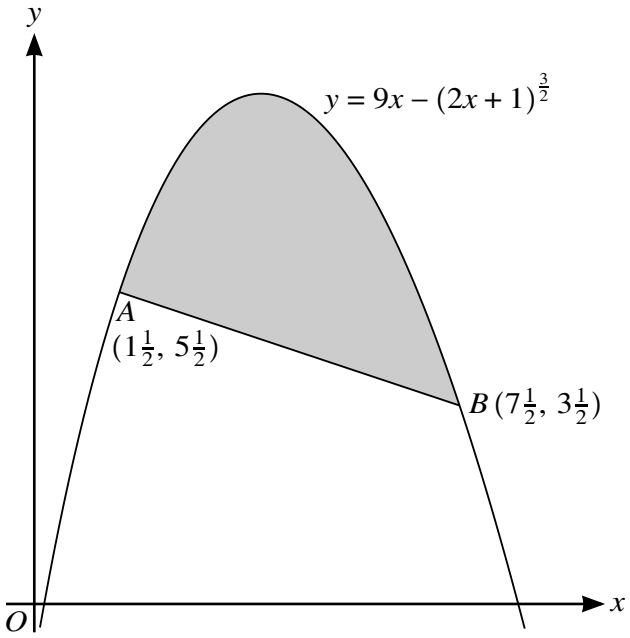

The diagram shows the points $A\left(1\frac{1}{2}, 5\frac{1}{2}\right)$ and $B\left(7\frac{1}{2}, 3\frac{1}{2}\right)$ lying on the curve with equation $y = 9x - (2x + 1)^{\frac{3}{2}}$.

(a) Find the coordinates of the maximum point of the curve. [4]

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(b) Verify that the line AB is the normal to the curve at A. [3]

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(c) Find the area of the shaded region. [5]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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